

# Diplomatic Dynamics of International Treaty Negotiations

T. DONG, R. ZENG, AND T. MA

**Abstract** - The negotiation process of international treaties is heavily determined by socioeconomic factors, such as non-developed countries needing resources from developed countries or developed countries outsourcing labor to non-developed countries. In this paper, we aim to analyze the dynamics in a five-compartment mathematical model using an epidemiological approach to understand treaty negotiation processes. We do so by categorizing them into different interest groups based on their attitudes towards the treaty (support, non-support, and withdrawal) as well as their socioeconomic statuses (developed and non-developed) and modeling the movement of countries between supporting, not supporting, and withdrawing. Using an epidemiological approach, we create parameters to measure the movement of countries from group to group. There are two equilibrium states, which reveal the conditions under which all countries would either support or oppose a treaty. We explore several hypothetical scenarios under which equilibria would occur, demonstrating the practical applications of our model. We further analyze the model's real life application through two case studies that are using the same interest groups towards an international matter involving the same type of socioeconomic classes: the first is on the Basel Convention's negotiation process and the second is on a non-traditional international treaty about carbon pricing.

**Keywords** : Basel Convention; carbon pricing; epidemiological approach; international treaty; negotiation dynamics

**Mathematics Subject Classification** (2010) : 91D30; 91F10; 92D30; 97M10

## 1 Introduction

In 1988, the Nigerian government discovered 18,000 tons of toxic waste illegally dumped in the small village of Koko by Italian businesses. Several people had already been poisoned by toxins leaching into the soil. The Koko Village incident raised public awareness of the international waste trade [16, 17]. In response, the United Nations (UN) organized the Basel Convention [11, 28], an environmental treaty that aimed to reduce and regulate the transboundary disposal of wastes. The strongest supporters and the first signers were the less-developed, waste-importing states. They were followed by many of the remaining global community over the next three years. However, some notable states such as the United States (US) ultimately withdrew from the treaty, directly causing its ineffectiveness today [11].



We define a *country coalition* as a group of countries sharing and advocating a specific interest. The case of the Basel Convention raises the question: how do different country coalitions change during the negotiation process of an international treaty?

The goal of this paper is to provide a mathematical framework to shed light on international negotiation dynamics. The model we present will provide a glimpse of the movement of countries over time. With this information, diplomats can craft more effective environmental treaties by choosing to include or exclude certain countries from parts of the negotiation process, based on how they might influence others [14]. They can also target countries who are more likely to be swayed. Additionally, the negotiation timeline might be shortened or lengthened, to take advantage of certain tipping points within the process that can be identified in our model. These strategic insights, when combined with effective policy, can make the difference between a successful and a failed treaty.

In the case of the Basel Convention, there is a coalition of waste-importing (supporters of treaty) countries, a coalition of waste-exporting (non-supporters) countries, and a coalition of countries who have withdrawn from the treaty. The scope and number of coalitions obviously differ from treaty to treaty. Whereas the dynamics within the Basel Convention can be separated into three main forces, other treaties appear more difficult to classify because a country can belong to different coalitions at the same time. For instance, the International Commission for the Conservation of Atlantic Tunas has multiple different coalitions of countries, each pushing for different limits on tuna catch while supporting for the treaty itself [31]. The variations in the number of interest groups and their goals can complicate any analysis of negotiation dynamics.

This paper examines a simplified version of negotiation dynamics by identifying three main coalitions based on support and whose scopes are determined by socioeconomic factors. Three coalitions is necessary for understanding negotiation dynamics, because in any international treaty, there will be countries who support the treaty, countries who oppose the treaty, and countries who withdraw from the treaty. Throughout the negotiation process, countries within each of these coalitions influence countries from other coalitions to adopt their position. Influence is heavily dependent on socioeconomic factors, as recent research into international relations reveals [13, 33]. Some countries choose to emulate economic, religious, and language peers. Others may prefer to appease trade partners. While the most economically powerful countries, like the US, are being emulated by other countries. In environmental cases especially, countries' economic statuses play the greatest role in determining whether or not they support the treaty [13, 33].

We define *influence* as the ability of a coalition to spread its ideas and values (e.g., support for a treaty) to countries in other coalitions, thus convincing them to change their positions. Many recent studies show that these ideas and values spread in a manner similar to epidemics in a population [5, 4, 15, 31]. Hence, we can divide the total population, all the countries in the world, into three compartments: supporters ( $S$ ), non-supporters



( $N$ ), and withdrawals ( $W$ ). Further, each of these compartments can be split according to their socio-economic status, and thus we can analyze the impact of these factors. We can model transitions between compartments, caused by inter-coalition influence, by a set of differential equations.

In terms of the voting process, there have been studies on how individuals have an influence and this can be extracted on an international level. In [20], Nickerson discusses how voting can be contagious and thus influences others to follow suit, or how people rely on close relations such as friends or family members to help obtain information that can affect their decisions [23]. The discussion in [6] explores strategic voting for citizens to elect officials that can have influence on international environmental debates that can be in favor of the voters. Once we have established which countries are grouped into  $S$ ,  $N$ , and  $W$ , then we can see which coalitions are being influenced by countries that are closely related to its members, such as US and Canada, or countries in the United Kingdom. There have been stochastic modeling studies on how influential individuals affect a core group [10, 21] such as lobbyists or activists recruiting voters. However, in applying the epidemiological framework, as inspired by [15], we can focus on the collective groups rather than a certain individual to model the behavior of how groups evolve in an international treaty proceedings.

The negotiation process encompasses the period in which diplomats from various countries are in direct contact at a formal meeting. We limit our time frame because first, the total number of countries in the world is very unlikely to change within such a short time span. Second, it is unlikely that a country's domestic politics and socioeconomic status will change drastically enough within this period to alter its stance on a treaty. Since this paper intends to focus on international dynamics, a short time period that allows us to focus solely on international factors is ideal.

In [15], the investigators study the development of third parties in the US and how they gain votes in elections. Their (third-party voting) model focuses on the voter population and groups them into five categories:  $M$  (party members),  $H$  (voters highly susceptible to third party ideology),  $L$  (voters barely susceptible to third party ideology),  $V_H$  (third party voting individuals deriving from  $H$ ), and  $V_L$  (third party voting individuals deriving from  $L$ ). There are influential parameters that quantify the rate at which some groups recruit new members from other groups. For example,  $\beta_1$  is the recruitment rate of  $H$  into  $V_H$  by individuals in  $V_H$ ,  $V_L$  and  $M$ . This model also considers the rate at which new people enter the population (i.e. those who have reached voting age) and leaving (i.e. death). Then, several differential equations are constructed to model the member changing rate of each group.

The way this third-party voting model quantifies the influential parameters via differential equations among groups is useful for the study of how countries influence each other during the negotiation process of a treaty. However, the way groups are categorized



is different from the international treaty negotiation process. For example, there is no equivalent of  $M$ , since the voting process for a treaty is not a periodically recurring process and thus there is no organization within those who support the treaty. In addition, the natural recruitment rate and attrition rate caused by the change in population is basically non-existent, as we assume in the short run, the total number of countries should not alter. With this in mind, we take the idea of influential factors and differential equations and create our Two-Track model that categorizes countries into different interest groups based on socioeconomic status and assigns new parameters to track how countries move from group to group.

In the Two-Track model, we divide countries into two socioeconomic groups: developed and less-developed. Further, we assume countries in each socioeconomic group move on a track with three options: to support, to not support, or to withdraw. In particular, countries can move back and forth between supporting and non-supporting, but once they withdraw they are unable to return to the negotiation. Therefore, we group countries into five coalitions: supporting developed countries, non-supporting developed countries, supporting less-developed countries, non-supporting less-developed countries, and withdrawn countries. This model is inspired by some previous studies and useful in studying the spread of ideas and values [15]. In terms of an epidemiological approach, we can establish that non-supporting and withdrawn countries are the “infected” groups and all others are “non-infected”. Moving into the non-supporting groups from the two support groups are, in a sense, capturing the behavior of new infections. Moving from non-supporting to withdrawn is the transitional movements of infected groups to infected groups.

Our paper is organized as follows: Section 2, we study a simplified One-Track model to show certain properties, including stability of equilibrium states. We then build upon the One-Track model in Section 3, where we analyze the Two-Track model’s equilibrium states and simulate hypothetical scenarios with parameters by choice. Finally, we discuss two case studies where we use the Two-Track model on the Basel Convention’s negotiation process (Section 4) and on a non-traditional international treaty of carbon pricing (Section 5).

## 2 The One-Track Model

First, let us consider the One-Track model to generate the discussion of how an epidemiological model can capture the behavior of international treaty negotiations. We will see that the one-track model is overly oversimplified and will need to build a more sophisticated model necessary to discuss the dynamics of treaty negotiations (in Section 3).



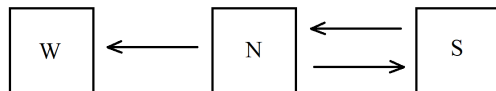


Figure 1: The One-Track Model Diagram. The arrows illustrate direction of movement between compartments.

We assume that ideas and values (e.g., support for a treaty) spread in a similar manner as epidemics in a population. This means that the population can be divided into several compartments and transitions between compartments can be modeled by a set of differential equations.

Taking inspiration from [15], we focus on country coalitions and that at any point during the negotiation of an international treaty, all countries can be classified as supporters, non-supporters, or withdrawn states. Supporters are defined as all the countries who, if negotiations were ended at some time, would sign the treaty. Non-supporters are defined as all who, if negotiations were ended at some time, would not sign the treaty. Withdrawn are defined as the countries who have left the negotiation process.

Let  $S$ ,  $N$ , and  $W$  be the variables to describe the number of countries who support the treaty, countries who do not support the treaty, and withdrawn countries respectively. As discussed in Section 1, the movement of countries from non-supporting status to withdrawn status is strictly unidirectional, while the movement between non-supporting status and supporting status is bidirectional. See Figure 1.

We assume that treaty negotiations occur over a relatively short period of time. Thus, the total number of countries ( $T > 0$ ) in the system remains constant ( $dT/dt = 0$ ). Domestic politics remain mostly stable within the short time frame. Hence, any changes to a country's position on a treaty can be attributed to international, rather than internal, influence. Countries that withdraw from negotiations will not return, which is why this movement is unidirectional.

In Section 1, we defined influence as the ability of a coalition to spread its ideas and values to countries in other coalitions and thus convincing movement between coalitions. Mathematically, "influence" in this model is represented by the function that characterizes the movement from one coalition to another. The goal of this analysis is not to identify what influences a country to move from coalition to coalition, whether it be the effectiveness of the contact that countries have with one another, the size of the country, etc. The goal is to quantify the movement of countries using the data that we have obtained to predict the movement of countries if the negotiations were to continue or if they were to end prematurely. This quantification of movement can account for several factors such as size of the country, effectiveness of discussion between countries, etc, however, we do not address causation of movement here as it is not the main goal of our study.



The movement from  $N$  to  $S$  is a function of the influence that all supporting countries exert on  $N$ :

$$\phi SN, \tag{1}$$

where  $0 \leq \phi \leq 1$  is the recruitment rate of  $N$  into  $S$ , a percentage of countries that a representative (or representatives) from countries in  $S$  comes in direct contact with representative (or representatives) from  $N$  that successfully convinces a country to change from the non-support to support coalition. Note that  $\phi = 0$  when there are no countries moving from non-support to support.

The number of countries that move from  $S$  to  $N$  is a function of the influence that all non-supporting countries and withdrawn countries exert on  $S_D$ :

$$(\beta N + \alpha' W)S, \tag{2}$$

where  $0 \leq \beta \leq 1$  is the attrition rate of countries in  $S$  caused by countries in  $N$ . Note that if  $\beta = 0$ , then none of the countries in the non-support coalition are trying to influence countries in the support coalition to change their position. If  $\beta < 1$ , then this means that not all of the countries in the non-support coalition are trying to influence countries in the support coalition to change their position. We can also note  $W$  has an augmentation factor ( $\alpha'$ ) to indicate the influence of countries who have withdrawn from negotiations can wield influence differently than countries that do not support, that is  $\alpha'$  may or may not be equal to  $\beta$ . If  $\alpha' = 0$ , then none of the withdrawn countries are trying to influence the other countries to withdraw. Note that  $\alpha'$  can be greater or less than  $\beta$ . For simplicity, we let  $\alpha = \alpha'/\beta$ , then (2) becomes:

$$\beta(N + \alpha W)S_D. \tag{3}$$

Finally, we can describe the movement of countries from non-supporting to withdrawn:

$$\gamma WN, \tag{4}$$

where  $0 \leq \gamma \leq 1$  is the attrition rate of  $N$  into  $W$  caused by countries in  $W$ . Similar to that of  $\phi$ , if  $\gamma = 0$ , then there are no countries moving to withdraw.

The following system of equations depicts the movement within the One-Track model depicted in Figure 1:

$$\frac{dS}{dt} = \phi SN - \beta(N + \alpha W)S, \tag{5}$$

$$\frac{dN}{dt} = \beta(N + \alpha W)S - \phi SN - \gamma WN, \tag{6}$$

$$\frac{dW}{dt} = \gamma WN, \tag{7}$$

$$T = S + N + W, \tag{8}$$

$$\frac{dT}{dt} = 0. \tag{9}$$



The movement of countries can be quantified by (1) - (4). For our analysis in Sections 2.1 and 2.2 we provide theoretical values for the parameters in equations (5) - (7).

## 2.1 Equilibria Analysis

We can determine the equilibria of the system by analyzing equations (5) - (9).

Setting (7) equal to 0, we can see that either  $W = 0$ ,  $N = 0$ , or both equal to 0. This leads to three types of equilibrium for the one-track model. The first is complete support, where we have all countries in the support coalition, and none in the non-support and withdrawn ( $N = W = 0$ ). The second is complete withdrawal, where we have all countries in the withdraw coalition, and none in the support and non-support ( $S = N = 0$ ). The last is complete non-support, where we have all countries in the non-support coalition, and none in the support and withdraw ( $S = W = 0$ ).

### 2.1.1 E1: Complete Support Equilibrium

According to equation (8), we can replace  $S$  by  $T - N - W$ , thus reducing the system of equations to the following:

$$\frac{dN}{dt} = \beta(N + \alpha W)(T - N - W) - \phi(T - N - W)N - \gamma WN, \quad (10)$$

$$\frac{dW}{dt} = \gamma WN. \quad (11)$$

To analyze the equilibria that depends on  $N$  and  $W$ , we compute the partial first derivatives (the Jacobian) with respect to  $N$  and  $W$ :

$$J(N, W) = \begin{pmatrix} N\phi - W\gamma - \beta S^* - \beta M + \phi S^* & N\phi - N\gamma - \beta M - \alpha\beta S^* \\ \gamma W & \gamma N \end{pmatrix},$$

where  $S^* = N - T + W$ ,  $M = N + W\alpha$ .

A treaty achieves complete support equilibrium at  $(0, 0)$ , when all countries support (and will likely sign) the treaty, that is when

$$J(0, 0) = \begin{pmatrix} (\beta - \phi)T & \alpha\beta \\ 0 & 0 \end{pmatrix}.$$

The eigenvalues of this Jacobian matrix are 0 and  $T(\beta - \phi)$ . Thus, when  $\beta < \phi$ , or when the attrition rate of supporting countries caused by non-supporting countries is less than the recruitment rate of non-supporting countries into the supporting coalition, the eigenvalues are non-positive and thus the equilibrium point  $(0, 0)$  is locally asymptotically stable.



### 2.1.2 E2: Complete Withdrawal Equilibrium

Similarly, we can replace  $W$  by  $T - N - S$ , reduce the system of equations to the following, and calculate its Jacobian with respect to  $S$  and  $N$ :

$$\frac{dS}{dt} = \phi SN - \beta(N + \alpha(T - N - S))S, \quad (12)$$

$$\frac{dN}{dt} = \beta(N + \alpha(T - N - S))S - \phi SN - \gamma(T - N - S)N, \quad (13)$$

$$J(S, N) = \begin{pmatrix} N\phi - \beta(N - \alpha W^*) + S\alpha\beta & S\phi + S\beta(\alpha - 1) \\ N\gamma - N\phi + \beta(N - \alpha W^*) - S\alpha\beta & N\gamma - S\phi + \gamma W^* - S\beta(\alpha - 1) \end{pmatrix},$$

where  $W^* = N + S - T$ .

A treaty achieves complete withdrawal equilibrium at  $(0,0)$ , when no country is in the negotiation process of the treaty:

$$J(0, 0) = \begin{pmatrix} -T\alpha\beta & 0 \\ -T\alpha\beta & -T\gamma \end{pmatrix}.$$

The eigenvalues of this Jacobian matrix are  $-T\gamma$  and  $-T\alpha\beta$ . With countries in  $W$  then  $\alpha$  and  $\gamma$  are positive. Since  $N = 0$ , then  $\beta = 0$ . The equilibrium point  $(0,0)$  is locally asymptotically stable.

### 2.1.3 E3: Complete Non-Support Equilibrium

We can replace  $N$  by  $T - S - W$ , thus reducing the system of equations to the following, and calculate its Jacobian with respect to  $S$  and  $W$ :

$$\frac{dS}{dt} = \phi(T - S - W)S - \beta(T - S - W + \alpha W)S, \quad (14)$$

$$\frac{dW}{dt} = \gamma W(T - S - W), \quad (15)$$

$$J(S, W) = \begin{pmatrix} S\beta - S\phi + \beta N^* - \beta W\alpha - \phi N^* & -S\phi - S\beta(\alpha - 1) \\ -\gamma W & -\gamma(N^* + W) \end{pmatrix},$$

where  $N^* = S - T + W$ .

A treaty achieves complete non-support equilibrium at  $(0,0)$ , when no countries support (nor will likely sign) the treaty, that is when

$$J(0, 0) = \begin{pmatrix} (\phi - \beta)T & 0 \\ 0 & \gamma T \end{pmatrix}.$$

The eigenvalues of this Jacobian matrix are  $T\gamma$  and  $-T(\beta - \phi)$ . Since  $W = 0$  there are no withdrawn countries and thus no influence from them, that is  $\gamma = 0$ . Thus, when





$\beta > \phi$ , that is when the attrition rate of support is higher than the recruitment rate of non-support to support, then the equilibrium point  $(0, 0)$  is locally asymptotically stable.

## 2.2 Expected Results

We assume that depending on the value of the parameters, different equilibria will be reached. From the analysis of eigenvalues, there are a few observations to be made.

If  $\beta < \phi$ , a complete support will be reached. Noting that the eigenvalues for the complete support case do not depend on  $\gamma$ , we will see that if  $\gamma > 0$  then (an almost) complete support will be reached with a few countries in the withdrawn coalition. If  $\gamma = 0$ , then all countries will be in support. We provide the following influential parameters for complete support in Figure 2:  $\beta = 0.049$ ;  $\phi = 0.15$ ;  $\alpha = 0.01$ ;  $\gamma = 0.02$ ;  $T = 195$ ; initial number of non-supporting countries ( $N$ ) is 185; initial number of withdrawn counties ( $W$ ) is 5; the time span is 1 year. Note that at the 1-year mark, because  $\gamma > 0$  and  $\beta < \phi$ , that  $N$  drops from 180 to 0 and  $W$  is staying steady at a low number, indicating that all other countries are in  $S$ .

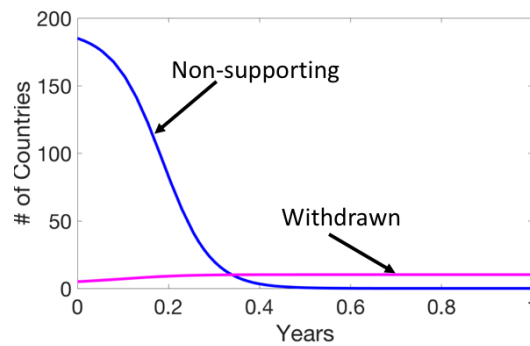


Figure 2: Complete Support. Here,  $N$  (Non-supporting, blue) approaches 0 and  $W$  (Withdrawn from negotiation, pink) stabilizes at a low level.

If  $\gamma > 0$  and  $\beta > \phi$ , then complete withdraw will be observed. We can see this in Figure 3, the influential parameters are:  $\beta = 0.075$ ;  $\phi = 0.015$ ;  $\alpha = 0.01$ ;  $\gamma = 0.02$ ;  $T = 195$ ; initial number of non-supporting countries ( $N$ ) is 90; initial number of supporting counties ( $S$ ) is 90; the time span is 5 years. We see that because  $\beta > \phi$  there is an increase in  $N$  and a decrease in  $S$ , but both coalitions decrease to 0 as negotiation is continuing to the 5-year mark.

If  $\gamma = 0$  and  $\beta > \phi$ , then complete non-support will be observed. This is evident from Figure 4, the influential parameters are:  $\beta = 0.075$ ;  $\phi = 0.015$ ;  $\alpha = 0.01$ ;  $\gamma = 0.00$ ;  $T = 195$ ; initial number of non-supporting countries ( $N$ ) is 90; initial number of supporting counties ( $S$ ) is 90; the time span is 1 year. Unlike the example with  $\gamma > 0$ , since  $\gamma = 0$ , there is no influence from the withdrawn coalition to attract non-supporters, thus  $N$  rises to its maximum.



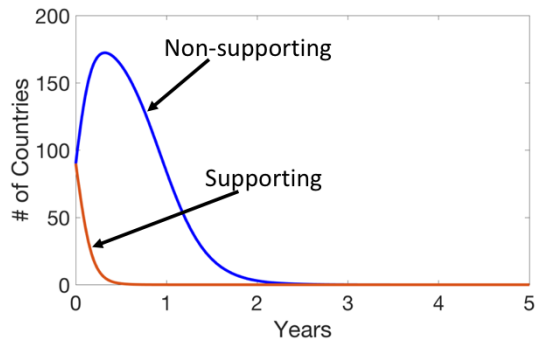


Figure 3: Complete Withdrawal.  $S$  (Supporting, orange) approaches 0 as countries turn against the treaty and enter  $N$  (Non-supporting, blue), causing the initial increase of  $N$ . Then  $N$  approaches 0 as countries further lose interest and withdraw from the negotiation process.

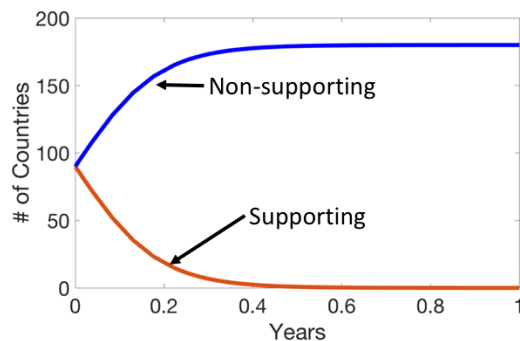


Figure 4: Complete Non-Support. Here,  $S$  (Supporting, orange) approaches 0 as countries turn against the treaty. They subsequently enter  $N$  (Non-supporting, blue) and reach an overwhelming majority.

### 3 The Two-Track Model

In the previous section, the One-Track Model only consisted of three compartments ( $S$ ,  $N$ , and  $W$ ). This simple representation was a great way to understand how countries are moving from one compartment to the other and how we can define the parameters to describe the movements.

However, to examine the influences of different socio-economic groups (developed or less-developed), we can roughly split the countries in supporting and non-supporting coalitions into sub-coalitions, depending on their status as developed or less-developed countries. We use the UN’s Human Development Index (HDI) to classify which 195 countries are socio-economic peers to each other. Specifically, 59 developed countries are those ranked in the “highly developed” category as designated by the UN [29]. Less-developed



countries are all of the remaining 136 countries. We can still assume that withdrawn countries exert a similar level of influence over both developed and less developed countries. This is because once they are absent from the negotiation table, they have substantially less power relevant to the treaty itself and thus do not influence any particular group more than the other. For future research, it would be interesting to split the withdrawn group into developed and less-developed countries to observe the different movements between coalitions.

We categorize countries into five groups and let  $S_D$ ,  $S_L$ ,  $N_D$ ,  $N_L$ , and  $W$  be the variables to describe the number of developed and less-developed countries who support the treaty, developed and less-developed countries who do not support the treaty, and withdrawn countries respectively. Countries in each socioeconomic group can move on a track with three options, namely supporting, non-supporting and withdrawal from one end of the spectrum to the other. As discussed in Section 1 and as we saw in Section 2, the movement of countries from non-supporting status to withdrawn status is strictly unidirectional, while the movement between non-supporting status and supporting status is bidirectional, see Figure 5.

The total number of countries ( $T$ ) in the system still remains constant ( $dT/dt = 0$ ). Also, countries that withdraw from negotiations will not return, which is why this movement is unidirectional.

The movement from  $N_D$  to  $S_D$  is a function of the influence that all supporting developed countries exert on  $N_D$ :

$$\phi_1(\tau_1 S_D + S_L)N_D, \tag{16}$$

where  $0 \leq \phi_1 \leq 1$  is the recruitment rate of  $N_D$  into  $S_D$ . Similar in the One-Track Model,  $\phi_1 = 0$  is when there are no countries moving from non-support to support. Note that  $S_D$  has an augmentation factor ( $\tau_1$ ) to indicate that countries among  $S_D$  can wield more influence than countries among  $S_L$ , meaning that  $\tau_1$  may or may not be equal to 1. The equation that describes the movement of  $N_L$  to  $S_L$  can be derived in a similar manner with respect to  $\phi_2$ , the recruitment rate of  $N_L$  to  $S_L$ , and  $\tau_2$ , the augmentation factor for  $S_L$ . If  $\tau_1 = 0$  or  $\tau_2 = 0$ , then none of the supporting developed or less-developed countries are trying to recruit or influence the non-supporting developed or less-developed countries to change their positions respectively.

The number of countries that move from  $S_D$  to  $N_D$  is a function of the influence that all non-supporting countries (both developed and less-developed) and withdrawn countries exert on  $S_D$ :

$$(\beta_1(N_D + N_L) + \alpha'_1 W)S_D, \tag{17}$$

where  $0 \leq \beta_1 \leq 1$  is the attrition rate of countries in  $S_D$  caused by countries in both  $N_D$  and  $N_L$ . If  $\beta_1 = 0$  then none of the countries in the non-support coalitions are trying to recruit or influence countries in  $S_D$  to change position. Note that  $W$  has an augmentation



factor ( $\alpha'_1$ ) to indicate the influence of countries who have withdrawn from negotiations can wield more influence than countries that do not support (from developed and less-developed), that is  $\alpha'_1$  may or may not be equal to  $\beta_1$ . For simplicity, we let  $\alpha_1 = \alpha'_1/\beta_1$ , then (17) becomes

$$\beta_1(N_D + N_L + \alpha_1 W)S_D. \quad (18)$$

The movement of  $S_L$  to  $N_L$  can be derived in a similar manner, with respect to  $\beta_2$ ,  $\alpha'_2$ , and  $\alpha_2$  as

$$\beta_2(N_D + N_L + \alpha_2 W)S_L. \quad (19)$$

In addition, since  $\alpha_1$  (and similarly  $\alpha_2$ ) are factors that indicate the influence of the countries that have withdrawn regardless of socio-economic status then we can let  $\alpha = \alpha_1 = \alpha_2$  be the factor to indicate the attrition rate of supporting countries to non-supporting countries by countries that have withdrawn from negotiations. If  $\alpha = 0$ , then none of the withdrawn countries are trying to influence other countries to change from support to non-support.

Finally, we can describe the movement of countries from non-supporting to withdrawn by

$$\gamma W(N_D + N_L), \quad (20)$$

where  $0 \leq \gamma \leq 1$  is the attrition rate of  $N_D$  and  $N_L$  into  $W$  caused by countries in  $W$ . Similar to the One-Track model,  $\gamma = 0$  indicates that none of the withdrawn countries are trying to influence countries to withdraw.

Similar to the One-Track Model, (16) - (20) quantifies the movement of countries for the Two-Track Model. We provide theoretical values for the parameters in equations (16) - (20) for our analysis in Sections 3.2. As we discuss specific examples in Section 4 and 5 we provide a way to calculate these parameters using obtained data.

The following system of equations describe our Two-Track model depicted in Figure 5:

$$\frac{dS_D}{dt} = \phi_1(\tau_1 S_D + S_L)N_D - \beta_1(N_D + N_L + \alpha W)S_D, \quad (21)$$

$$\frac{dS_L}{dt} = \phi_2(\tau_2 S_L + S_D)N_L - \beta_2(N_L + N_D + \alpha W)S_L, \quad (22)$$

$$\frac{dN_D}{dt} = \beta_1(N_D + N_L + \alpha W)S_D - \phi_1(\tau_1 S_D + S_L)N_D - \gamma W N_D, \quad (23)$$

$$\frac{dN_L}{dt} = \beta_2(N_L + N_D + \alpha W)S_L - \phi_2(\tau_2 S_L + S_D)N_L - \gamma W N_L, \quad (24)$$

$$\frac{dW}{dt} = \gamma W(N_D + N_L), \quad (25)$$

$$T = S_D + S_L + N_D + N_L + W, \quad (26)$$

$$\frac{dT}{dt} = 0. \quad (27)$$



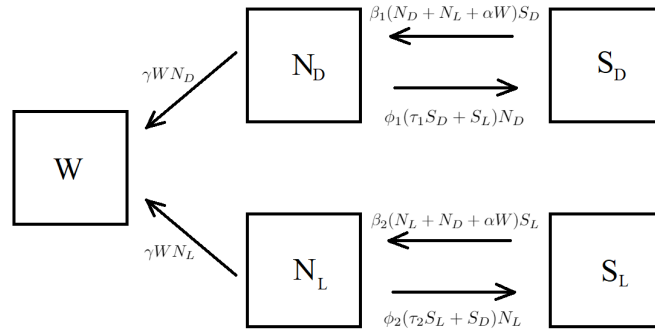


Figure 5: The Two Track Model Diagram. The arrows illustrate direction of movement between compartments.

### 3.1 Analysis of Two Track Model

We now turn to the stability analysis of the Two-Track Model. Similar to the One-Track Model, we will be analyzing the equilibria of the system.

#### 3.1.1 Steady-states

When  $W = 0$  we have the following steady-states:

$$S_1 = (S_D, S_L, 0, 0, 0), \quad (28)$$

$$S_2 = \left( \frac{\phi_1 S_L}{\beta_1 - \phi_1 \tau_1}, S_L, N_D, 0, 0 \right), \quad (29)$$

$$S_3 = \left( S_D, \frac{\phi_2 S_D}{\beta_2 - \phi_2 \tau_2}, 0, N_L, 0 \right), \quad (30)$$

$$S_4 = (0, 0, N_D, N_L, 0) \quad (31)$$

$$S_5 = \left( S_D, 0, \frac{\beta_1 N_L}{\phi_1 \tau_1 - \beta_1}, N_L, 0 \right), \quad (32)$$

$$S_6 = \left( 0, S_L, N_D, \frac{\beta_2 N_D}{\phi_2 \tau_2 - \beta_2}, 0 \right), \quad (33)$$

$$S_7 = (S_D^*, S_L, N_D^*, N_L^*, 0). \quad (34)$$

With stability analysis, we can perturb from the state of  $S_1$  and see through numerical simulations that  $S_1$  is unstable. Note that  $S_2$  is a solution only when  $\beta_2 = 0$  and  $\beta_1 > \phi_1 \tau_1$ . Similarly,  $S_3$  is a solution only when  $\beta_1 = 0$  and  $\beta_2 > \phi_2 \tau_2$ . By perturbing from the states and observing numerically, both  $S_2$  and  $S_3$  are unstable solutions. Similarly,  $S_4$  is an unstable solution. Also note that  $S_5$  is a solution when  $\phi_2 = 0$  and  $\phi_1 \tau_1 > \beta_1$ . Similarly,  $S_6$  is a solution when  $\phi_1 = 0$  and  $\phi_2 \tau_2 > \beta_2$ . Both  $S_5$  and  $S_6$  are semi-stable solutions from numerical observations.



Finally,  $S_7$  is the solution when there are countries in all groups except  $W$ , and

$$S_D^* = S_L R,$$

$$N_D^* = \frac{(T - S_D^* - S_L)\beta_1 S_D^*}{\phi_1(\tau_1 S_D^* + S_L)},$$

$$N_L^* = N_D^* \frac{\phi_1(\tau_1 S_D^* + S_L) - \beta_1 S_D^*}{\beta_1 S_D^*},$$

where  $R$  is the positive root of  $ax^2 + bx + c$  and  $a = \phi_2(\phi_1\tau_1 - \beta_1)$ ,  $b = \tau_2 a + \phi_1(\phi_2 - \beta_2\tau_1)$ ,  $c = \phi_1(\phi_2\tau_2 - \beta_2)$ .

We should note that  $S_7$  has its own class of solutions. For example, when  $a = 0$ , that is when  $\phi_1\tau_1 = \beta_1$ , we choose  $b$  and  $c$  values such that  $bc < 0$ , otherwise this would mean that  $S_D^* < 0$ . Therefore, when observing the steady state solution,  $S_7$  only has meaning when the choice of parameters  $\beta_1$ ,  $\beta_2$ ,  $\phi_1$ ,  $\phi_2$ ,  $\tau_1$ , and  $\tau_2$  make  $R$  a real positive number. There are four classes of solutions for when  $R$  is a real positive number. The first is when  $a = 0$  and  $bc < 0$  as we have described. The second is when  $b^2 - 4ac = 0$ , and  $b < 0$ , this root has multiplicity 2. The two other classes of solutions are:

$$S_{D,i}^* = S_L R_i,$$

$$N_{D,i}^* = \frac{(T - S_{D,i}^* - S_L)\beta_1 S_{D,i}^*}{\phi_1(\tau_1 S_{D,i}^* + S_L)},$$

$$N_{L,i}^* = N_{D,i}^* \frac{\phi_1(\tau_1 S_{D,i}^* + S_L) - \beta_1 S_{D,i}^*}{\beta_1 S_{D,i}^*},$$

for  $i = \{1, 2\}$  where  $R_i$  is one of the two positive distinct real roots of multiplicity 1 of  $ax^2 + bx + c$ . Each of these solutions are unstable through numerical observations.

When  $W \neq 0$ , we only have one steady-state:

$$W_1 = (0, 0, 0, 0, W), \tag{35}$$

which is stable through numerical observations.

### 3.2 Simulated examples

In this section, we simulate a few scenarios to show complete withdrawal from a treaty, an example of a successful treaty, and how time can play a role in the negotiation process.

In Figure 6, we simulate the complete withdrawal equilibrium of the two track model. Suppose there are 29 supporting developed countries ( $S_D$ ), 68 supporting less-developed countries ( $S_L$ ), 29 non-supporting developed countries ( $N_D$ ), 68 non-supporting less-developed countries ( $N_L$ ), and 1 withdrawn country ( $W$ ). Note that  $dW/dt$  (equation



(25)) is non-zero when both  $\gamma$  and  $W$  is not zero. That is, should there be no countries in  $W$  to recruit in the first place, the group will never exist.

We define  $t = 0$  as the beginning of negotiations,  $t = 1$  as the end of negotiations, and  $t > 1$  describes what would happen if negotiations were to continue.

In addition, we use the following parameters:  $\beta_1 = 0.055$ ;  $\phi_1 = 0.015$ ;  $\alpha = 0.025$ ;  $\beta_2 = 0.075$ ;  $\phi_2 = 0.005$ ;  $\gamma = 0.02$ ;  $\tau_1 = \tau_2 = 1.5$ ;  $T = 195$ . These initial test figures are derived from studying the Basel Convention and carbon pricing case studies, which we will explain in greater detail in Sections 4 and 5. Figure 6 is the plot of the simulation with the parameters we have indicated. The number of countries supporting the treaty of both groups ( $S_D$  in blue,  $S_L$  in orange) decrease, corresponding to the initial increase of number of countries that do not support the treaty ( $N_D$  in yellow,  $N_L$  in purple). After  $t = 0.75$ , the two groups that do not support the treaty start losing members as well, indicating that these countries have entered the withdrawn group. Hence the complete withdrawal state is reached in the 5-year period.

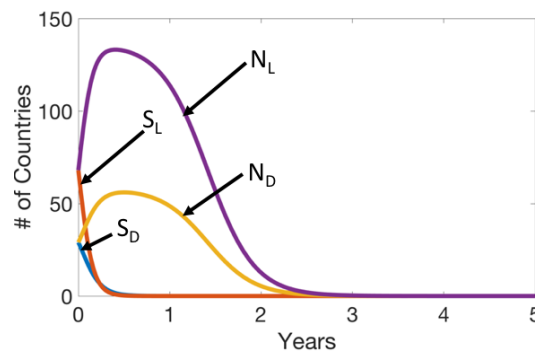


Figure 6: Complete Withdrawal for two track model. Blue:  $S_D$ , Orange:  $S_L$ , Yellow:  $N_D$ , Purple:  $N_L$ ;

Next, we have Figure 7 from the simulation with the following parameters:  $\beta_1 = 0.022$ ;  $\phi_1 = 0.015$ ;  $\alpha = 0.025$ ;  $\beta_2 = 0.002$ ;  $\phi_2 = 0.005$ ;  $\gamma = 0.02$ ;  $\tau_1 = \tau_2 = 1.5$ ;  $T = 195$ . The initial conditions again are:  $S_D = 29$ ,  $S_L = 68$ ,  $N_D = 29$ ,  $N_L = 68$ , and  $W = 1$ . The parameters are obtained in a fashion similar to the complete withdrawal scenario. In this simulation,  $\beta_2 < \phi_2$ , which can be understood as the recruiting influence of  $S_L$  being greater than that of  $N_L$ . As a result,  $S_L$  (orange line) is steadily increasing, corresponding to the drop in  $N_L$  (purple line). On the other hand,  $\beta_1 > \phi_1$ , which means the recruiting influence of  $S_D$  is less than  $N_D$ . This is evident from the initial drop in  $S_D$  (blue line) and initial increase in  $N_D$  (yellow line). However, as time goes on, the greater increase of  $S_L$  begins to cancel out the effect of  $\beta_1$  and  $S_D$  steadily rises, leading to the eventual pass of the treaty (Figure 7).

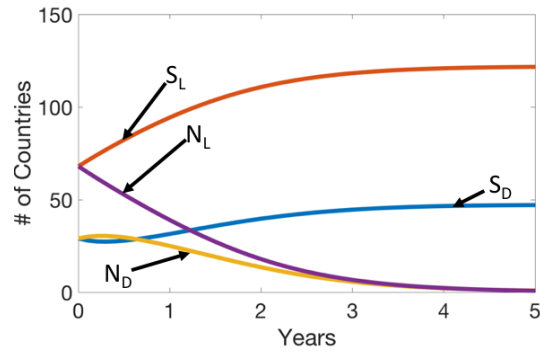


Figure 7: Treaty Success for two track model. Blue:  $S_D$ , Orange:  $S_L$ , Yellow:  $N_D$ , Purple:  $N_L$ ;

Finally, in Figure 8, we modify the parameters from Figure 7 to observe that the outcome of the model depends on the time when the negotiation process ends. The parameters are:  $\beta_1 = 0.08$ ;  $\phi_1 = 0.03$ ;  $\alpha = 0.025$ ;  $\beta_2 = 0.002$ ;  $\phi_2 = 0.005$ ;  $\gamma = 0.02$ ;  $\tau_1 = \tau_2 = 1.5$ ;  $T = 195$ . The initial conditions are :  $S_D = 29$ ,  $S_L = 68$ ,  $N_D = 29$ ,  $N_L = 68$ . Assuming that negotiation process ends at the  $t = 1$  year mark, we can see from Figure 8 how  $S_D$  (blue),  $S_L$  (orange),  $N_D$  (yellow), and  $N_L$  (purple) behave. If the negotiation ends around  $t = 2$  or after, which can be understood as if the negotiation process lasting twice as long as the originally planned time ( $t = 1$ ), the outcome will be that the treaty passes. However, if the negotiation process ends before that, for example at  $t = 0.3$ , which means if the negotiation process ends a third of the way through, there are more non-supporting ones (approximately 101 non-supporting versus 92 supporting), and therefore the treaty does not pass. The time-sensitivity in certain scenarios is worth noticing for political leaders and can mean the success or failure of a treaty [24, 30].

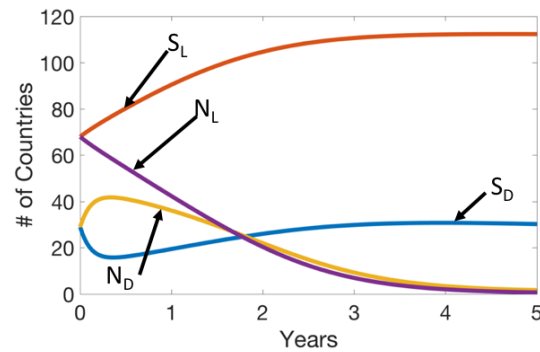


Figure 8: A negotiation process where being cut off at different time will yield different results. Blue:  $S_D$ , Orange:  $S_L$ , Yellow:  $N_D$ , Purple:  $N_L$ .





## 4 Case Study 1: The Basel Convention

We now return to the example we discussed in the introduction, the Basel Convention. The Basel Convention is an excellent case study for several reasons. First, the vast majority of decisions were reached early in the negotiation process. We will observe that if negotiations were to continue beyond this period that the negotiation process would have yielded a different outcome, similar to Figure 8. Second, the the total number of parties to the Convention remains constant at 140 states, as our model necessitates. Third, there is a clear socio-economic (developed and less-developed) divide present between waste-importers (supporters the Convention) and waste-exporters (opposers the Convention).

We used data on country ratification of the Basel Convention from 1989 to 1992. Countries who had ratified it were considered supporters, countries who had yet to ratify it were considered opposers, and countries who had left the process entirely were considered withdrawn. The socio-economic country groupings were determined by each country's 1990 UN HDI ranking. The top 58 countries fell in the UN's "highly developed" category, and thus were categorized as "developed" for the purposes of our study [28]. The remaining countries were categorized as "less developed" in our study. We should note that only 140 factions were represented in the negotiation process. Thus not all 58 developed-countries participated nor all less-developed countries participated.

From these assumptions, we found the following number of countries in the negotiation process:

Year	$S_D$	$S_L$	$N_D$	$N_L$	W
1989	22	9	33	74	1
1992	20	11	36	71	2

We had 2 countries move from  $S_D$  to  $N_D$ , 0 countries move from  $S_L$  to  $N_L$ , 0 countries move from  $N_D$  to  $S_D$ , 2 countries move from  $N_L$  to  $S_L$ , and 1 country move from  $N_L$  to W. For the purposes of our study, we arbitrarily input 0.01 as the number of countries that moves from  $S_L$  to  $N_L$  and from  $N_D$  to  $S_D$  to account for the possibility of uncaptured trends within a small population.

We manipulated equations (21) through (27) to yield the following:

$$\beta_1 = \frac{\text{Number of } S_D \text{ converted to } N_D}{\Delta t} \frac{1}{S_D(N_D + N_L + \alpha W)} \quad (36)$$

$$\beta_2 = \frac{\text{Number of } S_L \text{ converted to } N_L}{\Delta t} \frac{1}{S_L(N_D + N_L + \alpha W)} \quad (37)$$

$$\phi_1 = \frac{\text{Number of } N_D \text{ converted to } S_D}{\Delta t} \frac{1}{N_D(\tau_1 S_D + S_L)} \quad (38)$$



$$\phi_2 = \frac{\text{Number of } N_L \text{ converted to } S_L}{\Delta t} \frac{1}{N_L(\tau_2 S_L + S_D)} \quad (39)$$

$$\gamma = \frac{\Delta W}{\Delta t} \frac{1}{(N_D + N_L)W} \quad (40)$$

Inputting the values we observed into these equations and adjusting the  $\alpha$ ,  $\tau_1$ , and  $\tau_2$  accordingly, we were able to calculate the parameters:

$\beta_1$	$\beta_2$	$\phi_1$	$\phi_2$	$\gamma$
0.0004244122	0.0000034582	0.0000032584	0.0002906132	0.0031152648

#### 4.1 Recreating the Negotiation Dynamics

Using the parameters that we have calculated above combined with other parameters that we control for, we recreate the negotiation dynamics to study the effect of how time can play a role in the our model.

We can see that the model is fairly accurate in recreating the negotiation dynamics from  $t = 0$  to  $t = 3$  in Figure 9. In addition, we were interested to see what would happen if the negotiations were to extend beyond the 3-year mark. In Figure 9, we can see at the 5-year mark, there was a shift for less developed countries from non-supporter to supporter status, and a shift for developed countries from supporter to non-supporter status. In the ten-year prediction (Figure 10), the number of non-supporting, less-developed countries drops sharply. Some of this can be explained by a shift to supporting status. However, many of these less-developed countries have also moved into withdrawn status. This could be due to a variety of factors: some may have lost faith in the effectiveness of the Basel Convention, or some may have vested economic interests in continuing the waste trade [1, 17]. We should note that we only have the start and end data points of this study to calculate the parameters and do not have further data to test the accuracy of the model as there were only two meetings recorded [11].



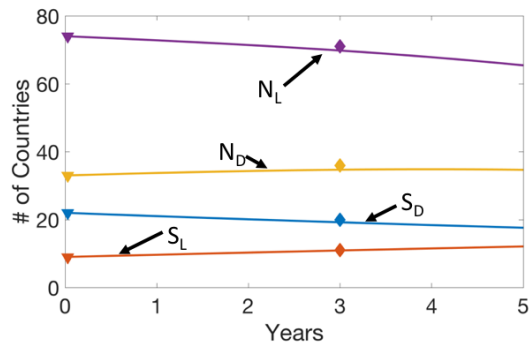


Figure 9: Simulation of the Basel Convention negotiation process when  $t = 5$ ,  $\tau_1 = \tau_2 = 1$ ,  $\alpha = 0.1$ . In this picture,  $S_D$  is in blue,  $S_L$  is in orange,  $N_D$  is in yellow,  $N_L$  is in purple. Triangle markers are the 1989 data points, diamond markers are the 1992 data points.

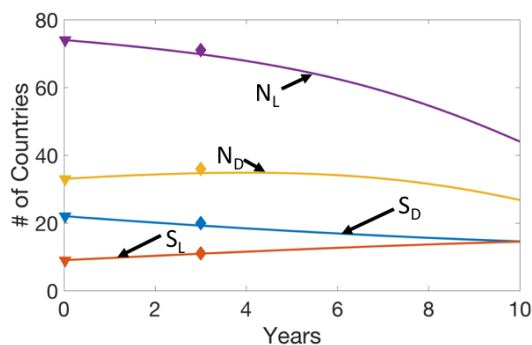


Figure 10: Simulation of the Basel Convention negotiation process when  $t = 10$ ,  $\tau_1 = \tau_2 = 1$ ,  $\alpha = 0.1$ . In this picture,  $S_D$  is in blue,  $S_L$  is in orange,  $N_D$  is in yellow,  $N_L$  is in purple. Triangle markers are the 1989 data points, diamond markers are the 1992 data points.

## 4.2 Testing Alternative Parameters

In Figures 11 and 12, we investigate whether intra-group influence of developed nations is stronger than that of less-developed nations. This could happen if less-developed nations had more factors to consider, such as greater economic dependence on other states, as well as their own domestic interests [7]. The number of supporting less-developed countries increases slightly as a result. However, the very small differences reveal the limited effect of intra-group influences on the overall outcome.

From the two simulations with alternative parameters, we can see that changing  $\tau_1$ ,  $\tau_2$ , and  $\alpha$  have limited effect in the outcome of the model. This seems to suggest that intra-group sympathy in the voting process, if it does exist, has trivial impact on the voting process. One explanation is most likely due to the size of the support coalition being smaller than that of the non-support. In this instance, the number of non-support



have a greater effect than the change of parameters. In addition, when the number of countries in  $W$  is small (approximately 0.67% of total countries), changing  $\alpha$  will not affect the outcome at all.

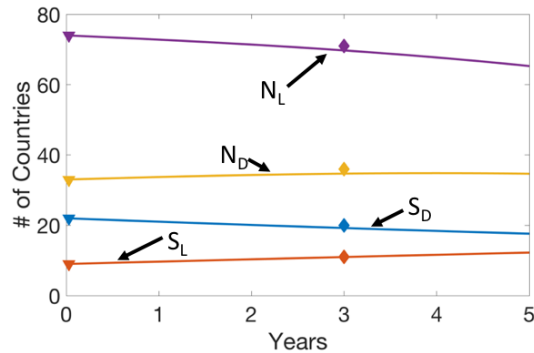


Figure 11: Simulation of the Basel Convention negotiation process when  $\tau_1 = 4$ ,  $\tau_2 = 2$ ,  $\alpha = 0.1$ , and other parameters are the same from previous simulation. In this picture,  $S_D$  is in blue,  $S_L$  is in orange,  $N_D$  is in yellow,  $N_L$  is in purple.

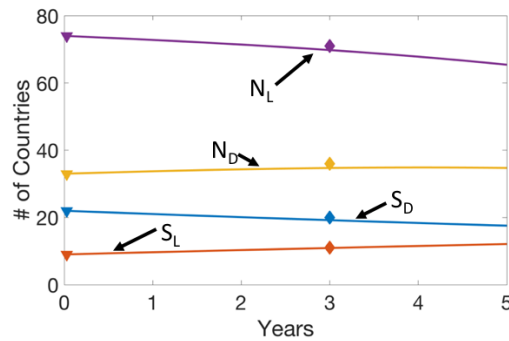


Figure 12: Simulation of the Basel Convention negotiation process when  $\tau_1 = 1$ ,  $\tau_2 = 1$ ,  $\alpha = 0.5$ .  $S_D$  is in blue,  $S_L$  is in orange,  $N_D$  is in yellow,  $N_L$  is in purple. Limited effect of the change of  $\alpha$  from 0.1 to 0.5.

## 5 Case Study 2: carbon pricing

We can further apply our model by studying the spread of carbon pricing at the state level. Carbon pricing presents an interesting case study in several ways. First and foremost, although it is not an international treaty itself, widespread support for carbon pricing policy mostly comes from treaties such as the Paris Agreement and the Kyoto Protocol [3, 22]. Thus, the spread of pricing policies will still be influenced by international negotiations, but with the benefit of accounting for multiple negotiation processes at once. Additionally, the greater timeline for carbon pricing will allow us to test how the model responds to the influence of domestic politics. Third, whereas the Basel Convention was



primarily driven by the interests of less-developed countries, carbon pricing has historically been driven by more developed countries due to the increased economic strain that the policy imposes. Investigating how developed countries' influence and less-developed countries' influence differs will be a valuable insight for our model.

We collected the data from the World Bank's "State and Trends of Carbon Pricing 2018." Countries who had implemented a carbon pricing scheme before a given date were considered "supporters" at that time. Countries who had not yet implemented a carbon pricing scheme before a given date were considered "non-supporters". By this definition there were no countries in the withdrawn coalition ( $W = 0$ ). The socio-economic country groupings were determined by each country's 2017 UN HDI ranking. The top 59 countries fell in the UN's "highly developed" category, and thus were categorized as "developed" for the purposes of our study. The remaining countries were categorized as "less-developed." We used the data collected within the first three years of carbon price policy to model state behavior.

The fact that this case study is not a traditional or official international treaty also means that the withdrawn coalition does not exist because countries are either implementing a pricing scheme (supporting) or not yet implemented (non-supporting) or renegotiating implementation (moving between support and non-support). This then simplifies the model and allow us to focus on the dynamics between support and non-support. With the absence of the withdrawn group, there is no movement from non-support coalitions to withdrawn coalitions ( $\gamma = 0$ ). Additionally, there is no factor of attrition of support to non-support caused by the withdrawn coalition ( $\alpha = 0$ ). The model is now dependent on the  $\phi_1$ ,  $\phi_2$ ,  $\beta_1$ ,  $\beta_2$ ,  $\tau_1$ , and  $\tau_2$  parameters.

From these assumptions, we found the following number of countries from the World Bank's "State and Trends of Carbon Pricing 2018" data:

Year	$S_D$	$S_L$	$N_D$	$N_L$
1990	2	0	57	136
1993	5	0	54	136
1996	6	0	53	136
1999	6	0	53	136
2002	7	0	52	136
2005	9	0	50	136
2008	12	0	47	136
2011	14	1	45	135
2014	19	2	40	134
2017	21	4	38	132
2019	24	5	35	131

Looking solely at the first three years, which we will be using to calculate parameters and predict the remainder of the table, 3 countries move from non-supporting, developed



status to supporting, developed status. Additionally, we added a rounding error of 0.5 to the number of countries moving from supporting, developed countries to non-supporting, developed countries. We also added the error to the number of less-developed countries moving between supporter and non-supporter status. This error came from the assumption that a majority of a state needs to support or reject a policy before change occurs, for instance if even half a state were inclined to support carbon pricing, it would be insufficient to appear as a change in the number of supporting countries.

$\beta_1$	$\beta_2$	$\phi_1$	$\phi_2$
0.0004317789292	0.008635578584	0.008354218881	0.00058356676

Using the above parameters calculated from equations (36) through (39), as well other parameters that we control for ( $\tau_1 = \tau_2 = 1, \alpha = 0$ ), we recreate the negotiation dynamics to predict future state behavior.

We can see from Figure 13, that the model closely portrays the negotiation dynamics of the carbon pricing study. Thus, not only can we use this model for international treaties, but we can use them for non-traditional international treaties. We should note that in the case of non-traditional treaties, that there are assumptions to be made that can characterize countries differently. For example, we used the first carbon pricing policies as the unofficial beginning of negotiations, since arguably, such a move would prompt other states to discuss the possibility of also implementing carbon prices. However, it is also possible that a cohort of states were independently catalyzed to implement their own carbon pricing schemes in the late 1980s. Thus, the initial year-to-year increases in carbon pricing adoption could simply be due to the differences in the time needed for each country to roll-out their own programs. Additionally, the non-treaty structure allows for the inclusion of more negotiation processes along with greater domestic influence, as stated earlier.

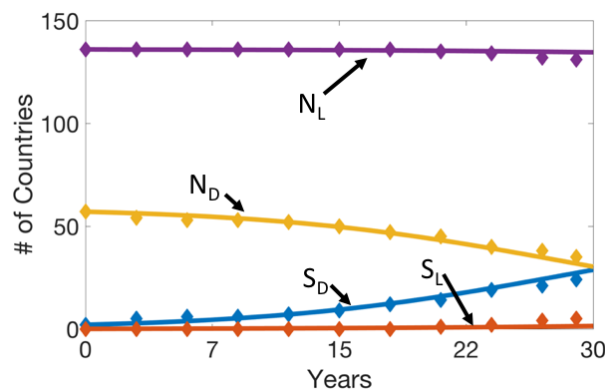


Figure 13: Simulation of the carbon pricing adoption process when  $t = 30, \tau_1 = 1, \tau_2 = 1$ . In this picture,  $S_D$  is in blue,  $S_L$  is in orange,  $N_D$  is in yellow, and  $N_L$  is in purple. Diamond markers are the data points collected from the World Bank’s “State and Trends of Carbon Pricing 2018”.



## 6 Discussion

From our results and analysis, we are able to draw several major conclusions. First, we can see that a large factor in the negotiation process is the sheer force of numbers as discussed in Section 4 when discussing alternative parameters. Changing the parameters  $\alpha$ ,  $\tau_1$ , and  $\tau_2$  had a minimal effect on whether or not countries shifted from category to category. A big portion of the effect comes from the size of each coalition. Second, in the negotiation process, intra-group influence is less important than other factors, and thus diplomats should focus on collaborating with those who share similar attitudes towards the treaty instead of trying to pull those dissidents that belong to the same socioeconomic group. Third, time-sensitivity exists in certain scenarios, where by controlling the ending time of the negotiation process, we can arrive at opposite outcomes. Therefore, if some diplomats can understand the movements of countries from previous negotiations using our model, they might initiate a motion to vote so the process will stop at a point where their interest is maximized. Ultimately, we can say that the epidemiological approach is roughly accurate in predicting the dynamics of international treaty negotiations. This does not simplify the complex process that diplomats have to go through in negotiating terms but we hope that this provides a bit of insight from a mathematical perspective into international negotiations.

We recognize that the case studies were limited in a number of ways, which we hope can be improved in future research. In the Basel case study, the data that was available were two data points that covered over three years and no further data to test the accuracy of the model. Thus the 5-year and 10-year predictions from this model as we discussed in Section 4.1 may not accurately reflect any behavior that countries have towards the treaty or with each other. We realize that the time period 1989-1992 was filled with other international developments that could have had a large impact on the outcome of the Basel Convention (for instance, the fall of the Berlin Wall [2, 18], the dissolution of the Soviet Union [8], and the birth of many new sovereign states). It is difficult to conclude that these events affected the outcome of the treaty itself or, in general, any events affecting the outcome of any particular treaty. If there had been more meetings to vote among the countries (more data points), then the model itself can provide a brief narrative into the behavior of the countries such as moving back and forth between supporting and non-supporting. From the carbon pricing study, although initially seemed to be less suited for our model, it nevertheless managed to capture accurate dynamics. Its success highlights the model's ability to account for domestically-driven policies where there are no cases of countries being withdrawn from negotiation.

For future studies of this model, we can study treaties that have more than two meetings to accurately predict their movements. Studies with large data size, such as the data obtained from the World Bank's "State and Trends of Carbon Pricing 2018", could account for the influence of external factors, such as domestic politics. However, we will note that we only concentrate on a relatively small sub-task of negotiation dynamics which is



quantifying movements of countries and predicting future movements. The grandiose task of identifying the causality of change in country coalitions using mathematics is beyond this research and can be the subject in future explorations. For other studies of this model, additional parameters could be introduced in order to account for varying interest groups' influences on a particular piece of policy. Modifying the model would make it possible to study smaller political ecosystems like the dynamics of a bilateral relationship, such as in the influence of American lobbying corporations and Korean Chaebols in the United States-South Korea relationship [27]. Other practical applications of our model include the study of peer pressure among teenagers (the spread of slang, drugs and games), the proliferation of meme culture, and the development of cancer cells [9, 19, 25].

Ultimately, the simplicity of the model is both a strength and a weakness; it allows for straightforward understandings of international treaty negotiations, but requires alteration in order to model more nuanced subjects.

## Acknowledgments

The authors of this paper would like to thank the referee who has put in an unbelievable amount of work and detail on suggestions for this paper through several iterations.

## References

- [1] A.A. Agbor, The Ineffectiveness and Inadequacies of International Instruments in Combatting and Ending the Transboundary Movement of Hazardous Wastes and Environmental Degradation in Africa, *African Journal of Legal Studies*, **9** (2016), 235–267.
- [2] G.M. Ahlfeldt, S.J. Redding, D.M. Sturm, N. Wolf, The economics of density: Evidence from the Berlin Wall, *Econometrica*, **83** (2015), 2127–2189.
- [3] A. Baranzini, J.C. Van den Bergh, S. Carattini, R.B. Howarth, E. Padilla, J. Roca, Carbon pricing in climate policy: seven reasons, complementary instruments, and political economy considerations. *Wiley Interdisciplinary Reviews: Climate Change*, **8** (2017), e462.
- [4] L.M. Bettencourt, A. Cintron-Arias, D.I. Kaiser, C. Castillo-Chavez, The power of a good idea: Quantitative modeling of the spread of ideas from epidemiological models, *Physica A: Statistical Mechanics and its Applications*, **364** (2006), 513–536.
- [5] F. Brauer, C. Castillo-Chavez, *Mathematical models in population biology and epidemiology*, Springer, 2001.
- [6] W. Buchholz, A. Haupt, W. Peters, International environmental agreements and strategic voting, *Scandinavian Journal of Economics*, **107** (2005), 175–195.
- [7] D. Colman, F.I. Nixson, *Economics of change in less developed countries*, Rowman & Littlefield, 1986.
- [8] L.S. Eastwood Jr., Secession: state practice and international law after the dissolution of the Soviet Union and Yugoslavia, *Duke J. Comp. & Int'l L.*, **3** (1992), 299.
- [9] G. Ghiaur, J. Gerber, R.J. Jones, Concise Review: Cancer stem cells and minimal residual disease, *Stem Cells*, **30**, (2012) 89–93
- [10] M. Granovetter, Threshold Models of Collective Behavior, *American journal of sociology*, **83** (1978), 489–515.





- [11] The Secretariat of the Basel Convention, History of the Negotiations of the Basel Convention, available online at the URL: <http://www.basel.int/TheConvention/Overview/History/Overview/tabid/3405/Default.aspx>
- [12] Human Development Ranking 1990, available online at the URL: <https://countryeconomy.com/hdi?year=1990>
- [13] B.J. Kinne, Network dynamics and the evolution of international cooperation, *American Political Science Review*, **107** (2013), 766–785.
- [14] B.J. Kinne, Dependent diplomacy: Signaling, strategy, and prestige in the diplomatic network, *International Studies Quarterly*, **58** (2014), 247–259.
- [15] C. Kribs, D.M. Romero, A. Mubayi, C. Orbe, An epidemiological approach to the spread of political third parties, Available at SSRN 1503124 (2011).
- [16] J. Krueger, The Basel Convention and the international trade in hazardous wastes. *Yearbook of international co-operation on environment and development 2001/02*, (2001), 43–51. Also available online at the URL: <http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.200.6196&rep=rep1&type=pdf>
- [17] K. Kummer, The international regulation of transboundary traffic in hazardous wastes: The 1989 Basel Convention, *International & Comparative Law Quarterly* **41** (1992), 530–562.
- [18] C. Lakner, B. Milanovic, Global income distribution: From the fall of the Berlin Wall to the Great Recession. *The World Bank*, 2013.
- [19] S. Meng, D. Tripathy, E.P. Frenkel, S. Shete, E.Z. Naftalis, J. Huth, P.D. Beitsch, M. Leitch, S. Hoover, D. Euhus, Circulating tumor cells in patients with breast cancer dormancy, *Clinical cancer research* **10**, (2004), 8152 – 8162.
- [20] D.W. Nickerson, Is voting contagious? Evidence from two field experiments, *American Political Science Review* **102** (2008), 49–57.
- [21] P.E. Oliver, G. Marwell, The paradox of group size in collective action: a theory of the critical mass. II., *American Sociological Review*, (1988), 1–8.
- [22] A. Pfaff, S. Kerr, R.F. Hughes, S. Lie, F.A. Sanchez-Azofeifa, D. Schimel, J. Tosi, V. Watson, The Kyoto protocol and payments for tropical forest: An interdisciplinary method for estimating carbon-offset supply and increasing the feasibility of a carbon market under the CDM, *Ecological Economics*, **35**, (2000), 203–221.
- [23] J.P. Robinson, Interpersonal influence in election campaigns: Two step-flow hypotheses, *Public Opinion Quarterly*, **40** (1976), 304–319.
- [24] J.W. Salacuse, Ten ways that culture affects negotiating style: Some survey results, *Negotiation journal*, **14** (1998), 221–240.
- [25] B. Simons-Morton, D.L. Haynie, A.D. Crump, P. Eitel, K.E. Saylor, Peer and Parent influences on smoking and drinking among early adolescents, *Health Education & Behavior* **28**, (2001), 95–107.
- [26] J. Sooknanan, D.M.G. Comissiong, When behaviour turns contagious: the use of deterministic epidemiological models in modeling social contagion phenomena, *Int. J. Dyn. Control*, **5** (2017), 1046–1050.
- [27] D. Steinberg, *Korean Attitudes Toward the United States: Changing Dynamics*, Routledge, 2015.
- [28] The Secretariat of the Basel Convention, Parties to the Basel Convention on the Control of Transboundary Movements of Hazardous wastes and their Disposal, available online at the URL: <http://www.basel.int/?tabid=4499>
- [29] United Nations Development Programme, Human development indices and indicators: 2018 Statistical update, *United Nations Development Programme South Africa*, 2018.
- [30] M. Watkins, Building momentum in negotiations: Time-related costs and action-forcing events, *Negotiation Journal*, **14** (1998), 241–256.



- [31] D.G. Webster, L. Gasser, *Adaptive governance: the dynamics of Atlantic fisheries management*, MIT Press, 2009.
- [32] The World Bank group, *World Bank Country and Lending Groups*, available online at the URL: <https://datahelpdesk.worldbank.org>
- [33] Y. Yamagata, J. Yang, J. Galaskiewicz, State power and diffusion processes in the ratification of global environmental treaties, *International Environmental Agreements: Politics, Law and Economics*, **17** (2017), 501–529.

*Tianhang Dong*  
Dartmouth College  
27 N. Main Street  
6188 Kemeny Hall  
Hanover, NH 03755  
E-mail: [Tianhang.Dong.19@dartmouth.edu](mailto:Tianhang.Dong.19@dartmouth.edu)

*Robin Zeng*  
Dartmouth College  
27 N. Main Street  
6188 Kemeny Hall  
Hanover, NH 03755  
E-mail: [Robin.Zeng.19@dartmouth.edu](mailto:Robin.Zeng.19@dartmouth.edu)

*Timmy Ma*  
Dartmouth College  
27 N. Main Street  
6188 Kemeny Hall  
Hanover, NH 03755  
E-mail: [Timmy.Ma@dartmouth.edu](mailto:Timmy.Ma@dartmouth.edu)

**Received:** July 11, 2019 **Accepted:** September 23, 2019  
**Communicated by Cynthia Flores**

